IEEE P802.11
Wireless LANs

Graphs, Nets, and Meshes

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Abstract

There are results in graph theory which may be instructive in considering the standardization of 802.11 ESS Meshes or in other 802.11 areas. In particular, there is reason to believe that most real world network node interconnect cardinality follows a power law distribution and incorporating this into simulation models will make them more realistic. This document provides some bibliographic pointers from Noel Chiappa and Jon Crowcroft, via Robert Moskowitz, reprinted here with permission.

The following material was kindly provided by Noel Chiappa via Robert Moskowitz:

There have been some really interesting developments in the last few years in the theory of random graphs.

The work on random graphs by Erdos (in the 50's) and Bollobas (in the 80's) focused on "classical" random graphs, where the probability of any particular arc E_{ij} (between nodes i and j) is a fixed p. This produced graphs where the distribution of the degree of the nodes (i.e. number of arcs coming to each node) was a Poisson bell-curve centered on a fairly small number.

Such graphs tended to have important differences from real Internet topologies, so the ability to apply theoretical results from this space was somewhat limited.

However, starting in 1999, with a paper by Albert-Laszlo Barabasi and Reka Albert, "Emergence of Scaling in Random Networks", a new sub-field of graph theory, the so-called "scale-free random graphs", where the distribution of node degree follows an declining exponential (where the exponent is a constant), with most nodes having a few links, but some having a large number. (In fact, if one plots the number of nodes of a particular degree versus the degree, on log-log paper, the result is a straight line – not surprising given the generating function.)

A large number of real networks (BGP AS's, actors, WWW, scientific paper references, etc) seem to follow the scale-free model quite closely. Theoretical results from the study of such random graphs (e.g. average shortest inter-node path length – which they, most confusingly, call "diameter", unlike the classical definition in graph theory) are thus of great import.

For those who are interested in finding out more, I can recommend the following sites which contain links to a large number of the most relevant papers:

Powerlaws: Hype or Revelation?
http://powerlaws.media.mit.edu/

Web Structure and Algorithms
http://www.math.cmu.edu/~af1p/WWW2004/WWW2004.html

From Random Graphs to Complex Networks
http://stat-www.berkeley.edu/users/aldous/Networks/

The following material was kindly provided by Jon Crowcroft via Robert Moskowitz:

The first small world measurement paper by the 3 Faloutsos brothers at SIGCOMM 99, was really the proper patient zero paper.

Drawing nice pictures of small world graphs seems to be a popular thing and one student at UCL did a bunch of it. See http://www.cybergeography.org/.

A number of people have looked at web node degree distributions – some of them physisiicts and some form a more database world. See for example
http://www.dcs.bbk.ac.uk/~mark/

There's some nice stuff on the truth of power law and small world nets by Sugih Jamin. See
http://topology.eecs.umich.edu/

The one I like most was by a student looking for economic models that expalined the emergence of the r-gaph structure. They worked out the number of factors involved when someone makes a decision to connect to the net. (Why not the nearest, or the most connected, or the cheapest or the most reliable, etc. etc.)
Internet Connectivity at the AS-level: An Optimization-Driven Modeling Approach, in Proceedings of ACM SIGCOMM Workshop on MoMeTools, 2003 (extended version, Tech Report UM-CSE-475-03).

Then there are geographical models of the internet (now known as internet coordinate systems). See work by Crovella et al. at BU which cites other work that was done at Intel and CMU:
http://www.icir.org/vern/imc-2003/papers/p320-tang1.pdf